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COMPARISON OF HIGUCHI, KATZ AND MULTIRESOLUTION BOX-COUNTING FRACTAL DIMENSION ALGORITHMS FOR EEG WAVEFORM SIGNALS BASED ON EVENT-RELATED POTENTIALS

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ABSTRACT

Obtaining information through the measurement of brain signals recorded during different processes or physiological conditions is important for developing computer interfaces that translate electrical brain signals to computer control commands. Electroencephalography (EEG) records the electrical activity of the brain in response to its receipt of different external stimuli (potential events). Analysis of these signals makes it possible to identify and distinguish specific states of physiological brain function. The Fractal Dimension has been used as a tool for biomedical waveform analysis, in particular to measure the complexity of time series generated by EEG. This paper aims to analyze a database (HeadIT) of biomedical time series obtained by EEG for which the fractal dimension will be obtained by the Higuchi, Katz and multiresolution box-counting methods, showing the relationship between the method for obtaining the fractal dimension and the physiological condition of the brain event-related potentials.

KEYWORDS: Fractal Dimension, Higuchi, Katz, multiresolution box-counting, EEG waveforms.

COMPARATIVO DE LOS ALGORITMOS DE DIMENSIÓN FRACTAL HIGUCHI, KATZ Y MULTIRESOLUCIÓN DE CONTEO DE CAJAS EN SEÑALES EEG BASADAS EN POTENCIALES RELACIONADOS POR EVENTOS

RESUMEN

La obtención de información por medio de la medición de señales registradas durante diferentes procesos o condiciones fisiológicas del cerebro es importante para poder desarrollar interfaces computacionales que traduzcan las señales eléctricas cerebrales a comandos computacionales de control. Un electroencefalograma (EEG) registra la actividad eléctrica del cerebro en respuesta al recibir diferentes estímulos externos (potenciales por eventos). El análisis de estas

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señales permite identificar y distinguir estados específicos de la función fisiológica del cerebro. La Dimensión Fractal se ha utilizado como una herramienta para el análisis de formas de ondas biomédicas, en particular se ha utilizado para determinar la medida de la complejidad en series de tiempo generadas por EEG. El presente documento pretende analizar la base de datos *HeadIT* de series de tiempo biomédicas obtenidas por EEG a las cuales se obtendrán la FD por medio de los métodos Higuchi, Katz y Multi-resolución de Conteo de Cajas, que muestre la relación entre el método para la obtención de la Dimensión Fractal y la condición fisiológica de la señal basada en Potenciales Cerebrales Relacionados por Eventos..

PALABRAS CLAVE: Dimensión Fractal, Higuchi, Katz, Multiresolución de Conteo de Cajas, señales EEG.

COMPARATIVO DOS ALGORITMOS DE DIMENSÃO FRACTAL HIGUCHI, KATZ E MULTI-RESOLUÇÃO DE CONTAR AS CAIXAS EM SINAIS EEG BASEADAS EM POTENCIAIS RELACIONADOS POR EVENTOS

RESUMO

A obtenção de informação por médio da medida de sinais registados durante diferentes processos ou condições fisiológicas do cérebro é importante para poder desenvolver interfaces computacionais que traduzam os sinais elétricos cerebrais a comandos computacionais de controle. Um eletroencefalograma (EEG) regista a atividade elétrica do cérebro em resposta ao receber diferentes estímulos externos (potenciais por eventos). A análise destes sinais permite identificar e distinguir estados específicos da função fisiológica do cérebro. A Dimensão Fractal utilizou-se como uma ferramenta para a análise de formas de ondas biomédicas, em particular utilizou-se para determinar a medida da complexidade em séries de tempo geradas por EEG. O presente documento pretende analisar séries de tempo biomédicas obtidas por EEG às quais obter-se-ão a FD por médio dos métodos Higuchi, Katz e Multi-resolução de Conteo de Caixas, que mostre a relação entre o método para a obtenção da Dimensão Fractal e a condição fisiológica do sinal baseado em Potenciais cerebrais relacionados por eventos.

PALAVRAS-CHAVE: Dimensão Fractal, Higuchi, Katz, Multi resolução da conta de Caixas, sinais EEG.

1. INTRODUCTION

Brain computer interfaces (BCIs) monitor the brain activity of the user and translate his or her “intentions” in the form of orders without activating a single peripheral muscle or nerve (Millán et al., 2014). For the development of BCI systems, it is necessary to find tools that make it possible to homogenize the physiological condition of the users to be able to bring said systems under a type of control based on the “intention” of the user. The electrical signals obtained through electroencephalography (EEG) are used to clinically evaluate brain activity. BCI systems interpret the physiological behavior of the brain (intention) through electrical event-related potentials (ERPs) to create computer commands that enable the development of electronic device control applications. In the following paper, the biomedical EEG signals obtained in response to external visual stimuli (visual evoked potential, or VEP) will be analyzed. Said VEPs are represented as time sequences (time series) for the electrical potentials obtained by way of the electrodes placed on the scalp.

One of the tools used for analyzing the EEG signals is the fractal dimension (FD), a term introduced by Mandelbrot (1983) that is applied to objects in space or fluctuations in time that possess some forms of self-similarity and cannot be described in a single scale of absolute measurement. FD refers to a non-whole number or a fractional dimension of an object.

We define (X, d) as a metric space where space X is a set of objects called points and d is a metric as a function $d : X \times X \rightarrow \mathbb{R}$, that measures the distance between a pair of points (x, y) in space X. We will consider the number $N(r)$ the number of maximum fixed-radius circles r necessary to completely cover X, $X \subseteq \mathbb{R}^2$. $N(r)$ and inversely proportional to r. We can say that

$$N(r) = \left(\frac{1}{r}\right)^{FD} \qquad (1)$$

when the value of $r \rightarrow 0$ and we can find the smallest number of closed radius areas r necessary to cover space X, meaning that the FD is defined by

$$FD = \lim_{r \rightarrow 0} \frac{\log(N(r))}{\log(1/r)} \qquad (2)$$

FD analysis is frequently used in biomedical signal processing, including EEG analysis, which has made it possible to study the dynamic chaos of the brain (Lutzenberger et al., 1995) and identify and distinguish specific states of its physiological functions (D. Easwaramoorthy and R. Uthayakumar, 2010). In particular, it has been used to measure the complexity of EEG signals (B. S. Raghavendra and D. N. Dutt, 2009).

FD analysis has also been used on many occasions in biomedical signal processing in the form of EEG analysis (Bachmann et al., 2013), (Baljekar and Patil, 2012), (Bojié et al., 2010), (Jevtić, and Paskaš, 2011), (Esteller et al., 2001), (Georgiev et al., 2009), (Harne, 2014), (Katz, 1988), (Khoa and Toi, 2012), (Loo et al., 2011), (Paramanathan and Uthayakumar, 2008), (Polychronaki et al., 2010), (Raghavendra and Dutt, 2009) and (Spasićm et al., 2011) as well as in a variety of aspects of systems engineering (Cervantes-De la Torre et al., 2013), (Gálvez et al., 2013), (Martins et al., 2012), (Millán et al., 2014), and (Perlingeiro et al., 2005). This paper focuses on experimental EEG-derived signals, and the algorithms proposed are those of Higuchi, Katz and the multiresolution box-counting (MRBC) method. Their results are widely applicable to any type of signal.

Various studies using EEG signals have employed FD algorithms: Polychronaki et al. (2010) for the detection of the start of an epileptic crisis; Easwaramoorthy and Uthayakumar(2010) used EEG signals to analyze brain activity during cognitive processes (reading, attention, memory, etc.); Loo et al. (2011) used EEG signals based on motor imagery for BCI systems; Bashashati et al.(2003) relied on FD methods to identify the control components of EEG signals in BCI systems; Esteller et al.(2001) and Raghavendra and Dutt (2010, 2009) used synthetic signals as datasets for the calculation of FD based on fractal behavior similar to that of EEG signals.

The relationship between the physiological condition of the EEG biomedical signals, based on ERPs, and the method for measuring the signal complexity will make it possible to show in a general sense how FD methods for signal analysis can be implemented in BCI systems. Here, one of the challenges is that the generalized condition of the users could be interpreted a certain way by a control device.In this paper the complexity of the biomedical EEG signals during short periods of time (fractograms) will be analyzed through calculation of the FD using the Higuchi, Katz and MRBC algorithms.

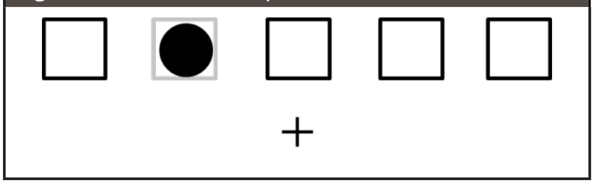
2. METHODOLOGY

2.1. Experimentation

The signals analyzed in this project were registered during a test called theFive Boxes Test, in which the signals obtained were based on VEP described by S. Makeig (1999). The study was conducted with 15 right-handed volunteers, 12 men and 3 women between 19 and 53 years of age with an average of 30 years, 12 with normal vision and 3 with corrected vision.

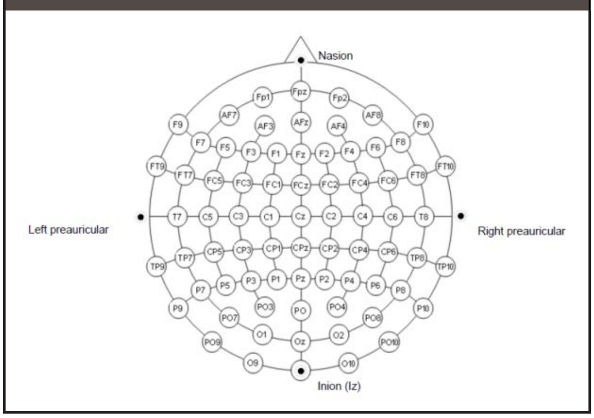
In the Five Boxes Test, the participants fixed their sight on a cross, above which five boxes were continuously exhibited (pictures) (**Figure 1**). In each test block of 76 seconds, one of the boxes (gray box) was a different color. This picture was randomly placed throughout the test periods. One series of circles were briefly presented in one of the five boxes in random order. The participant was asked to respond by pressing a button as quickly as possible each time a disc appeared in one of the boxes.

Figure 1. Visual stimulus pattern for the Five Boxes Test



EEG data were obtained from 29 scalp electrodes mounted on a standard electrode mesh (ElectroCap, Inc.) based on the International 10-20 Modified Combinatorial Nomenclature (MCN) system as shown in **Figure 2**, and from two periocular electrodes placed below the right eye and in the outer corner of the left. The data were sampled at 256Hz with an analog bandpass filter (BPF) of 0.01-50 Hz. Subsequently, responses were digitally filtered with a low-pass filter (LPF) below 40 Hz prior to analysis (S. Makeig et al., 2004).

Figure 2. International 10–20 Electrode System featuring MCN



2.2. Data selection

The data (obtained from the Human Electrophysiology, Anatomic Data, and Integrated Tools, HeadIT; belonging toSwartz Center for Computational Neuroscience(SCCN) of the University of California San Diego, United States of America; founded and developed by the U.S. National Institutes of Health grants R01-MH084819(Makeig, Grethe PIs) and R01-NS047293(Makeig PI))are organized into sessions, each representing the implementation of the Five Boxes Test with a random number of events, and each study subject doing one or more sessions. The information obtained during the experiment generates an EEG of 32 channels that were obtained through EEGLABv7.1.3.13b software, freely distributable under the GNU GPL free software license, and developed by Delorme and Makeig (2004).

1. Wave signals and time series

The EEG signals obtained from a set of electrodes fixed on the cerebral cortex are irregular time series represented as waveforms. We will consider the waveform a discrete time series in the following manner:

$$S = \{s_1, s_2, \ldots s_N \qquad (3)$$

where N represents the total points in the series and s the successive values of the EEG. The graph of the series is represented as $s_i = (x_i, y_i)$, i = 1,2, ... N, where x_i are the values of the abscissae and y_i the values of the ordinates (**Figure 3**). In the waveforms of the time series $s_i = t_i$, they increase monotonically at the point in time when the wave is shown (Dubravka et al., 2011).

FD algorithms allow us to interpret the chaotic behavior in irregular time series, represented as wave signals, and discriminate patterns based on similarity. (P. Paramanathan and R. Uthayakumar, 2007).

Figure 3. Representation of the waveform as a time series where the X axis represents the points of the series and the Y axis represents the amplitude of the signal

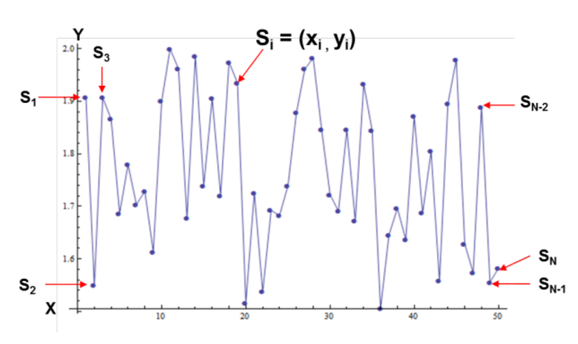
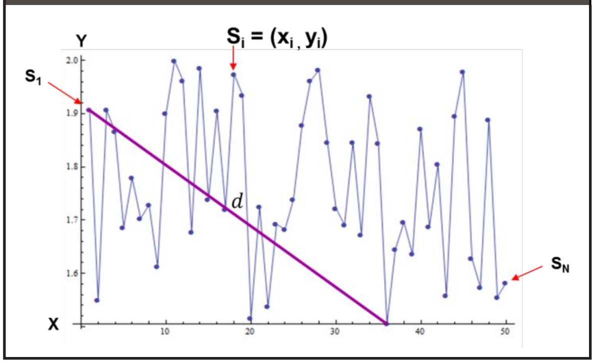


Figure 4. *d* represents the planar extension of the time series *S*



2. *Katz algorithm*

The calculation of FD proposed by Katz (1988) is described as the ratio of the length of curve *L*, calculated as the sum of the Euclidean distances between two successive points, divided by the maximum distance *d* of any point in the in the frame in question from the first point (M. Katz, 1988). We can interpret it as the ratio of the total length of the curve compared to the straight line corresponding to the maximum Euclidean distance from the first point. The algorithm defines FD as

$$FD = \frac{\log_{10}(L)}{\log_{10}(d)} \qquad (4)$$

where *L* is the total longitude of the curve or the sum of the Euclidean distances between successive points

$$L = \sum_{i=1}^{N} dist\,(s_i, s_{i+1}),\ i = 1, \ldots N{-}1 \qquad (5)$$

and *d* is the diameter (or planar extension) of the curve, meaning the distance between the first point in the sequence and the furthest point in the sequence (**Figure 4**), *d* can be expressed as

$$d = max\,\{dist\,(s_1, s_i),\ i = 1, \ldots N\} \qquad (6)$$

Katz proposed normalizing *d* and *L* by the length of the middle stage or the mean distance between successive points, $a = \frac{L}{N}$, where *N* is the number of steps in the curve. Thus, (4) becomes

$$FD = \frac{\log_{10}\left(\frac{L}{a}\right)}{\log_{10}\left(\frac{d}{a}\right)} = \frac{\log_{10}(N)}{\log_{10}\left(\frac{d}{L}\right) + \log_{10}(N)} \qquad (7)$$

3. *Higuchi algorithm*

For the Higuchi algorithm, *S* is considered the time series to be analyzed. The algorithm consists of forming new waveforms, subsequences of *S*, by iterative selection samples that differ in their origin point *m* and their delay factor or discrete time interval between points *k* (delay). First, we select the maximum delay factor, k_{max} Thus, for each delay factor *k*, where *k* varies from 1 to k_{max}, we form *k*'s new time series, S_k^m, defined as

$$S_k^m = \left\{ s_m, s_{(m+k)}, S_{(m+2k)}, \ldots, s_{(m+\lfloor a \rfloor k)} \right\} \qquad (8)$$

where $a = \frac{N-m}{k}$, $m = 1,2,\ldots, k$; $k = 1,\ldots, k_{max}$, *m* and *k* are whole positives.

For example, if $k = 3$ and $N = 100$, the constructed time series are defined as

$S_3^1 = s_1,\ s_4,\ s_7,\ s_{10}, s_{97},\ \ s_{100}$

$S_3^2 = s_2,\ s_5,\ s_8,\ s_{11}, s_{98},$

$S_3^3 = s_3,\ s_6,\ s_9,\ s_{12}, s_{99},$

for each constructed time series S_k^m its mean length L_k^m is defined by

$$L_k^m = \frac{\sum_{i=1}^{\lfloor a \rfloor} \left| s_{(m+ik)} - s_{(m+(i-1)k)} \right| (N-1)}{\lfloor a \rfloor k} \quad (9)$$

where N is the total length of the data sequence S and $(N-1)/(\lfloor a \rfloor k)$ is the *normalizing constant* for the length of the subsequence.

We then calculate the average length of the curve for each k, $\langle L_k \rangle$ as the mean value of the L_k^m of the k subsequences, which is defined by

$$\langle L_k \rangle = \frac{1}{k} \sum_{m=1}^{k} L_k^m \quad (10)$$

The average length $\langle L_k \rangle$ of the series S is obtained by the average of all the lengths L_k^m of the k subsequences. This procedure is repeated for each range of k from 1 to k_{max} (G. E. Polychronaki et al, 2010).

If $\langle L_k \rangle \propto k^{-FD}$, then the curve is a fractal with dimension FD, in which case the graph log_{10} ($\langle L_k \rangle$) *vs* $log_{10}(k)$ must approximate a straight line with a slope equal to –FD, whereby FD can be calculated using a linear least squares approximation (G. E. Polychronakiet al., 2010).

4. Multiresolution box-counting algorithm

The MRBC algorithmis based on the space-filling properties of a curve. The curve is covered with a set of objects of the same area or boxes (in this case square boxes). A size is determined for the area of each object, and the minimum number of boxes necessary to cover the curve is counted. As the size of the boxes approaches zero, the total area covered by the boxes will converge to the desired size of the curve.

This algorithm seeks to obtain the FD for various box sizes and make a linear fit to a graph $log_{10}(N(r))$ on $log_{10}(r)$. The slope of the least squares line is taken as an estimation of the FD of the curve (B. S. Raghavendra, y D. N. Dutt, 2010).

We consider S, with a frequency f_s. Each point s_i in the sequence is represented as (x_i, y_i), $i = 1, \ldots N$. Likewise, the signal is represented by a period (resolution) $r = \frac{1}{f_s}$.

To start the MRBC, two points on the curve are taken to represent the signal s_i, $s_{(i+1)}$. The time interval between the points is given by

$$dt = x_{(i+1)} - x_i = \frac{1}{f_s} \quad (11)$$

the height between the points is

$$h = y_{(i+1)} - y_i \quad (12)$$

(**Figure 5**) the size of the box considered to cover the two points is dt, and the number of boxes required to cover the points is

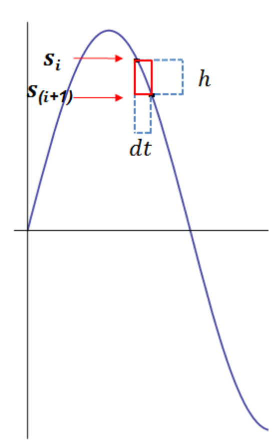
$$b_i = \left\lceil \frac{|h|}{dt} \right\rceil \quad (13)$$

the total of resolution boxes r required to cover the curve is calculated by

$$B_r = \sum_{i=0}^{N-1} b_i, \; i = 1, \ldots N-1 \quad (14)$$

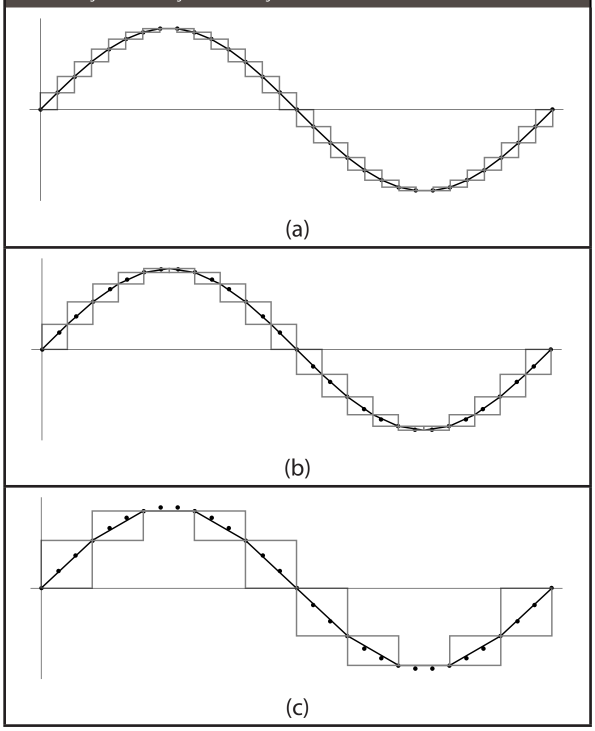
and the procedure repeats for all the points on the curve.

Figure 5. Identification of the elements s_i, $s_{(i+1)}$, h, dt



For the next step of the MRBC, the repetition of the aforementioned procedure for multiple resolutions is considered to be $r = \frac{1}{f_s}, \frac{2}{f_s}, \ldots \frac{R}{f_s}$, where $\frac{R}{f_s}$ is the maximum resolution that can be observed in the curve (**Figure 6**) (B. S. Raghavendra, y D. N. Dutt, 2010).

Figure 6. MRBC approximation for a sine wave. a) $r = \frac{1}{f_s}$ b) $r = \frac{2}{f_s}$ c) $r = \frac{3}{f_s}$



3. RESULTS

For this project a sample of 10 sessions from the work of Makeig was taken, each of which was randomly selected. The signals were converted to text format, and 10-second fractograms within the 500-510 second range were generated for analysis (**Figure 7**).

The algorithms were implemented using the Wolfram Languagein Mathematica V9.0.1.0. N = 2561 was considered the total number of points in the series for all the algorithms that were implemented. For the Higuchi algorithm m= 2 and k = $\left[\frac{N}{2}\right]$ were considered, and for the MRBC method, R = 1000 was considered. **Table 1** shows the results obtained from the FD, and **Table 2** shows the statistical variations obtained in each algorithm.

TABLE 1. FRACTAL DIMENSION COMPARISON BY STUDY SUBJECT

Study subject	Higuchi	Katz	MRBC
140	1.00216	1.29736	1.01828
142	1.00222	1.46924	1.01964
319	1.00222	1.30689	1.01837
325	1.00228	1.53963	1.02011
309	1.00241	1.73597	1.02121
138	1.00222	1.34862	1.01873
318	1.00227	1.39007	1.01906
317	1.00229	1.36287	1.01884
314	1.00226	1.27650	1.01809
131	1.00197	1.16174	1.01685

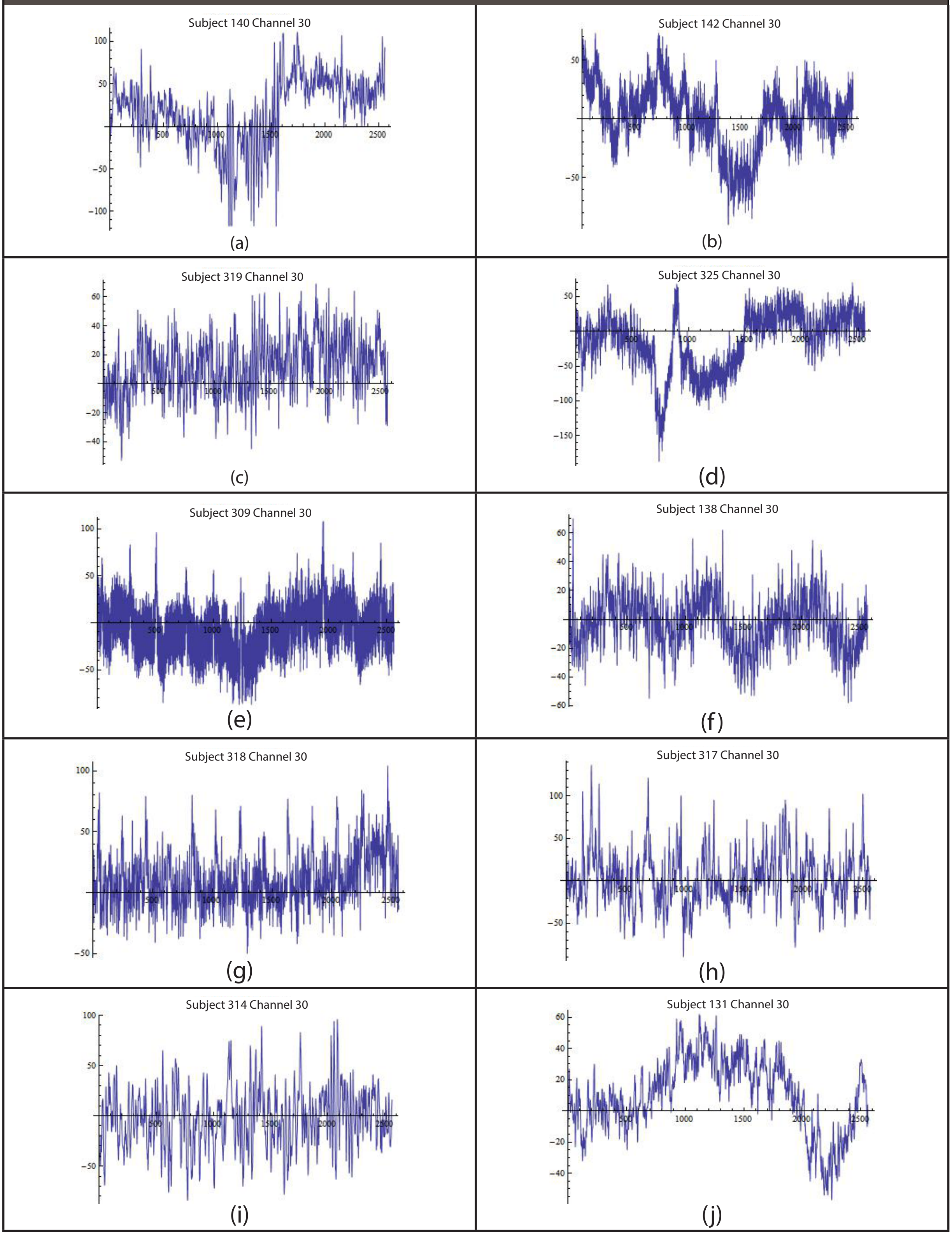
TABLE 2. COMPARISON OF FD VARIATIONS

Method	Variance	Standard deviation
Higuchi	1.26444×10^{-8}	0.000112448
Katz	0.0447103	0.160303
MRBC	1.43628×10^{-6}	0.00119845

4. CONCLUSIONS

Calculating the FD allowed us to determine the complexity of the EEG signals obtained. In the obtained results in **Table 2**, we can see that the FD in the Higuchi algorithm is maintained within the range of $1.000 < FD < 1.0003$ in the Katz algorithm it is maintained within $1.0 < FD < 2.0$ and in the MRBC method within $1.00 < FD < 1.03$. The variation of the FD in the Higuchi and MRBC algorithms is sufficiently small to be able to consider the FD as just one, with the Higuchi algorithm being a good option chiefly for implementation in BCI systems.

Figure 7. Signals corresponding to channel 30 (O1) during the 500-510s range



5. DISCUSSION

In the experiment carried out for this paper, the ERPs were randomized for each participant. In order to have a better vision of the behavior of the FD algorithms in the EEG signals, the randomness of the events must be decreased just like the size of the fractograms, being smaller due to having a duration of less than one second. There are other algorithms for FD calculation—Bouligand-Minkowski, Grassberger-Proccacia, the Hurst exponent, among others—which need to be implemented and compared to have a more complete view. The works that follow will focus on the implementation of the algorithms presented in this paper under more controlled experimental conditions with regard to VEP and fractograms in the one-second range.Additionally, the Bouligand-Minkowski, Grassberger-Proccacia, and Hurst exponent algorithms will be implemented for comparison.

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